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Effect of variable viscosity in the presence of variable wall temperature on condensation on a horizontal tube

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Abstract—Variable viscosity solutions have been obtained taking acccount of both radial and tangential temperature variation. Solutions have been obtained for ethylene glycol, which has a strong dependence of viscosity on temperature, and for both free and forced convection. For uniform surface temperature, constant-property solutions, with an appropriate reference temperature, give accurate results for the mean heat-transfer coefficient, even where the viscosity varies across the condensate film by a factor of around 10. Constant-property solutions are much less accurate in the presence of appreciable circumferential wall temperature variation. The theoretical results are compared with heat-transfer measurements for condensation of ethylene glycol.

INTRODUCTION

EFFECTS OF variable properties on laminar condensation heat transfer have been considered by several investigators [1–6]. It has been variously suggested that a reference temperature of the form:

$$T^* = xT_v + (1-x)\bar{T}_w, \tag{1}$$

where x depends on the condensing fluid, should be used when determining condensate properties for use in uniform property solutions. Somewhat different values of x are quoted, but values near 1/3 are frequently found and this figure is often used.

The temperature dependence of viscosity is much stronger than that of density and thermal conductivity and consequently viscosity variation is the dominant variable-property effect when calculating the heat transfer. In experiments with ethylene glycol, carried out in the course of the present work, the viscosity varied within the condensate film in some cases by factors exceeding 10, while conductivity and density never varied by more than 10%.

In previous theoretical investigations of the effect of variable properties, the tube wall temperature was taken to be uniform, so that account was taken only of viscosity variation radially across the film. In practice the wall temperature varies appreciably around the tube [7–11]. It is also seen from these investigations that the local wall temperature distribution around the tube surface can be approximated quite closely by:

$$T_w = a\cos\phi + b, \tag{2}$$

when the temperature difference across the condensate film is given by:

$$\Delta T = \overline{\Delta T}(1 - A\cos\phi), \tag{3}$$

where $0 \leqslant A \leqslant 1$.

The temperature variation around the tube is stronger (i.e. larger value of A) when the ratio of the condensate thermal resistance to that of the tube wall and coolant is smaller.

Uniform property solutions, with a vapor–surface temperature difference given by equation (3), have been obtained by Memory and Rose [10] and Memory *et al.* [11] for free and forced convection condensation, respectively. In the present work solutions are obtained for variable wall temperature and *with variable viscosity*. Comparisons are made with uniform wall temperature, uniform property (at the reference temperature given by equation (1) with $x = 1/3$) and with variable property, uniform wall temperature solutions.

NOMENCLATURE

A	constant, see equation (3)
a	constant, see equation (2)
b	constant, see equation (2)
D	defined in equation (13)
d	diameter of tube
E	defined in equation (12)
$f_1(\phi)$	function defined in equation (10)
$f_2(\phi)$	function defined in equation (11)
g	specific force of gravity
h_{fg}	specific enthalpy of evaporation
k	thermal conductivity of condensate
m	local condensation mass flux
q	local heat flux
$\bar{q}$	mean heat flux
T	local temperature in condensate film
T_v	vapor temperature
T_w	local wall temperature
$\bar{T}_w$	mean wall temperature
T^*	reference temperature, see equations (1) and (17)
u	tangential velocity in condensate film
U_∞	vapor approach velocity
x	constant, see equation (1)
y	radial distance from tube surface.

Greek symbols

δ	local condensate film thickness
δ_0	condensate film thickness at $\phi = 0$
ΔT	local vapor-to-surface temperature difference
$\overline{\Delta T}$	mean vapor-to-surface temperature difference
μ	viscosity of condensate
ρ	density of condensate
ϕ	angle from top of tube.

THEORY

We consider the case of downward vapor flow, where the motion of the condensate is affected both by gravity and vapor shear stress, and when the tube wall temperature has a cosine variation. For the purpose of investigating the effect of variable viscosity, it was considered adequate to adopt the asymptotic shear stress approximation used by Shekriladze and Gomelauri [12] and so assume potential flow outside the vapor boundary-layer. Results for the special cases of free and forced convection only, with a uniform wall temperature, can readily be obtained.

As in the Shekriladze and Gomelauri [12] and Nusselt [13] solutions, for the condensate film we neglect the inertia terms in the momentum balance and the convection terms in the energy balance. The approximation of radial conduction, as used by Honda and Fujii [7], Memory and Rose [10] and Memory *et al.* [11], is also adopted. Density and thermal conductivity of the condensate are taken to be uniform and pressure gradients in the condensate film, resulting both from gravity and vapor flow, are neglected.

With the above assumptions a momentum balance for the condensate film yields:

$$\mu \frac{\partial u}{\partial y} = \rho g(\delta - y)\sin\phi + 2mU_\infty \sin\phi. \tag{4}$$

A mass balance for an element of the film gives:

$$m = \frac{2\rho}{d}\frac{\mathrm{d}}{\mathrm{d}\phi}\int_0^\delta u\,\mathrm{d}y$$

$$= \frac{2\rho}{d}\frac{\mathrm{d}}{\mathrm{d}\phi}\left[\int_0^\delta \left\{\rho g \sin\phi \int_0^y \frac{(\delta - y)}{\mu(y)}\mathrm{d}y + 2mU_\infty \sin\phi \int_0^y \frac{1}{\mu(y)}\mathrm{d}y\right\}\mathrm{d}y\right], \tag{5}$$

where u has been obtained by integration of equation (4).

Conduction across the film gives:

$$m = \frac{k\Delta T}{h_{fg}\delta}. \tag{6}$$

The dependence of vapor–wall temperature difference on angle is given by equation (3) so that equation (5) may be written as:

$$\frac{k\overline{\Delta T}(1 - A\cos\phi)}{h_{fg}\delta} = \frac{2\rho}{d}\frac{\mathrm{d}}{\mathrm{d}\phi}\left[\int_0^\delta \left\{\rho g \sin\phi \int_0^y \frac{(\delta - y)}{\mu(y)}\mathrm{d}y + \frac{2k\overline{\Delta T}(1 - A\cos\phi)U_\infty \sin\phi}{h_{fg}\delta}\int_0^y \frac{1}{\mu(y)}\mathrm{d}y\right\}\mathrm{d}y\right]. \tag{7}$$

Differentiation of equation (7) with respect to ϕ, together with the symmetry condition

$$\frac{\mathrm{d}\delta}{\mathrm{d}\phi} = 0 \quad \text{when} \quad \phi = 0, \tag{8}$$

gives the value of δ at the top of the tube:

$$\delta_0 = \frac{\{1 - 4Ef_2(0)\}(1 - A)}{2Df_1(0)}, \tag{9}$$

where

$$f_1(\phi) = \int_0^\delta \left\{\int_0^y \frac{(\delta - y)}{\mu(y)}\mathrm{d}y\right\}\mathrm{d}y, \tag{10}$$

$$f_2(\phi) = \int_0^\delta \left\{ \int_0^y \frac{1}{\mu(y)} \mathrm{d}y \right\} \mathrm{d}y, \tag{11}$$

$$E = \frac{U_\infty \rho}{d}, \tag{12}$$

$$D = \frac{\rho^2 g h_{fg}}{k d \overline{\Delta T}}. \tag{13}$$

Equation (7), which may be written as:

$$\frac{(1-A\cos\phi)}{\delta} = 2D\frac{\mathrm{d}}{\mathrm{d}\phi}[\sin\phi f_1(\phi)] + 4E\frac{\mathrm{d}}{\mathrm{d}\phi}\left[\frac{\sin\phi(1-A\cos\phi)}{\delta}f_2(\phi)\right], \tag{14}$$

may then be solved numerically with the initial condition, equation (9), and a suitable equation for the dependence of condensate viscosity on temperature.

Numerical solutions to equation (14) have been obtained for ethylene glycol, which has a strong dependence of viscosity on temperature. The viscosity–temperature relationship for this fluid [14] was represented by:

$$\left(\frac{\mu}{\mathrm{kg\,m^{-1}\,s^{-1}}}\right) = \exp\left[-11.0179 + 1744/\left(\frac{T}{\mathrm{K}}\right) - 280\,335/\left(\frac{T}{\mathrm{K}}\right)^2 + 1.12661\times10^8/\left(\frac{T}{\mathrm{K}}\right)^3\right]. \tag{15}$$

Equation (15), with the temperature distribution across the condensate film:

$$T = \frac{y}{\delta}\Delta T + T_w, \tag{16}$$

gives the required dependence of viscosity on y.

The density and thermal conductivity of the condensate were taken at the reference temperature:

$$T^* = \tfrac{2}{3}\bar{T}_w + \tfrac{1}{3}T_v. \tag{17}$$

Calculated local heat flux distributions around the tube are shown in Fig. 1 for free convection [second term on the right-hand side of equation (14) omitted] and in Fig. 2 for forced convection [first term on the right-hand side of equation (14) omitted]. Relatively large values of $\overline{\Delta T}$ (100 K for free convection and 60 K for forced convection) and vapor velocity (100 m s^{-1}) for the forced convection case were used. Results were obtained both for uniform ΔT ($A = 0, \Delta T = \overline{\Delta T}$) and with strong dependence of ΔT on angle ($A = 0.6$). For all cases, results were also obtained with constant viscosity equal to the value calculated at the reference temperature, equation (17).

The strong effect of wall temperature variation on the heat flux distribution, discussed earlier for the uniform viscosity case by Memory and Rose [10] for free convection, and by Memory *et al.* [11] for forced convection, is clearly seen.

For the uniform ΔT case (uniform wall temperature), the uniform viscosity solutions do not differ greatly from those obtained when the viscosity is allowed to vary across the condensate film despite the fact that the viscosity changes across the film by factors up to 10. It is interesting to note that the variable viscosity solution gives slightly lower heat fluxes for

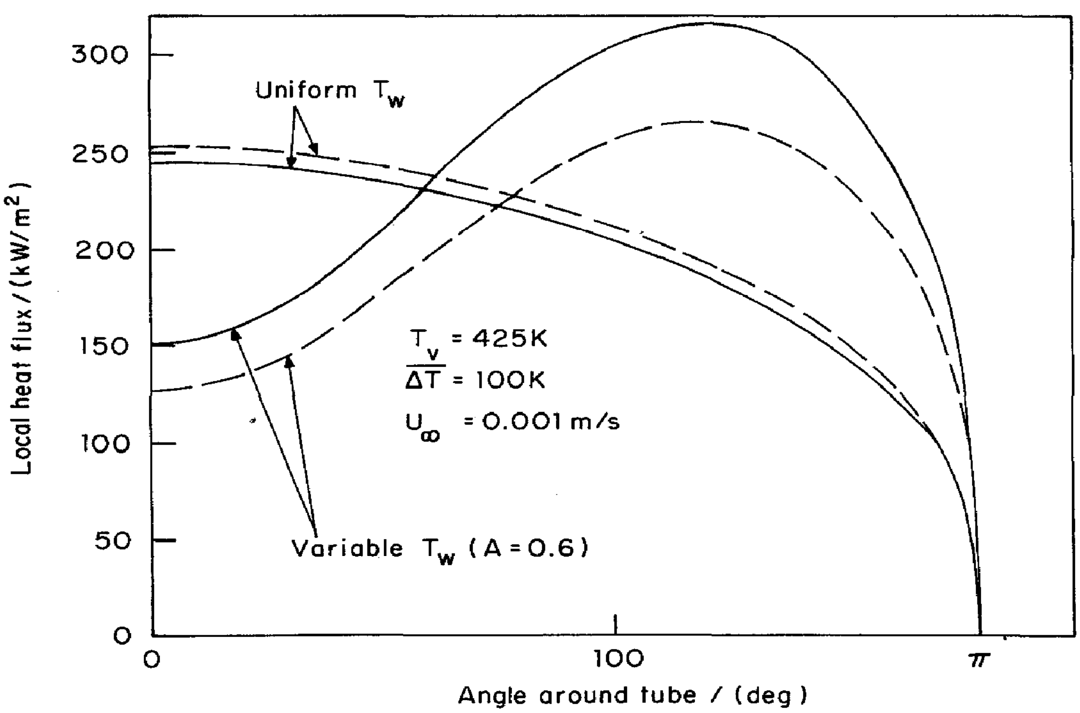

FIG. 1. Free convection condensation–heat flux distributions for uniform and variable wall temperature and condensate viscosity: (——) variable viscosity; and (- - -) uniform viscosity at T^* ($d = 12.5$ mm).

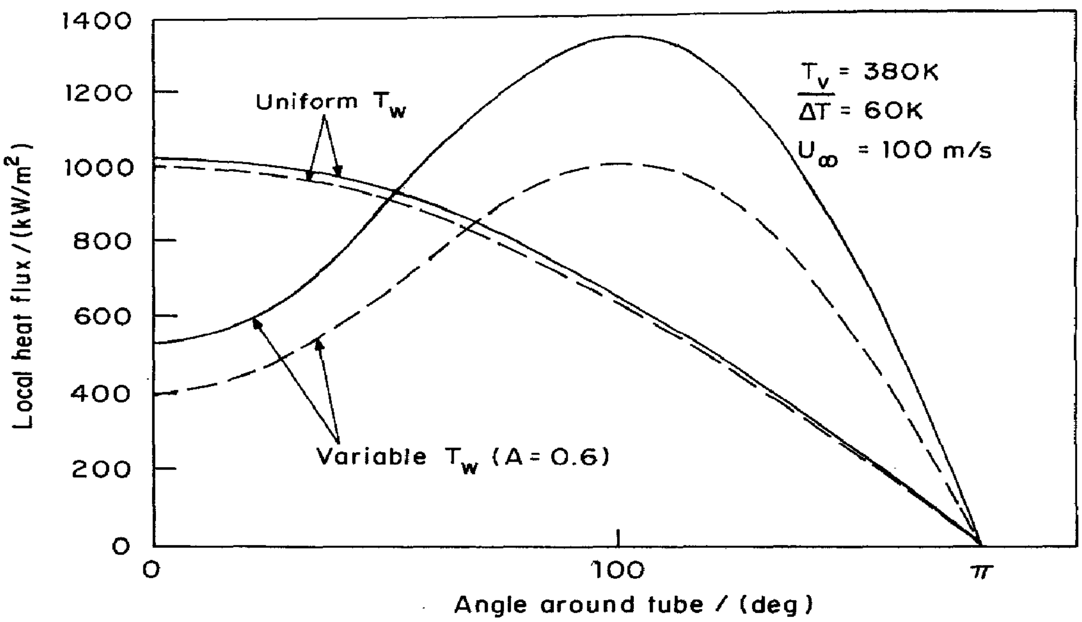

FIG. 2. Forced convection condensation–heat flux distributions for uniform and variable wall temperature and condensate viscosity: (——) variable viscosity; and (---) uniform viscosity at T^* ($d = 12.5$ mm).

free convection and slightly higher heat fluxes over the upper part of the tube for forced convection.

The effect of variable viscosity is more evident for the case of variable wall temperature. For both free and forced convection condensation the variable viscosity solutions give higher heat fluxes.

Representative results giving the dependence of mean heat flux:

$$\bar{q} = \frac{1}{\pi}\int_0^\pi q\,\mathrm{d}\phi, \tag{18}$$

on mean temperature difference, $\overline{\Delta T}$, are given in Table 1 for the cases of free and forced convection. Also included are results obtained when assuming uniform surface temperature and uniform viscosity taken at the reference temperature, equation (17).

It may be seen from Table 1 that viscosity variation affects the mean heat flux most strongly at the highest values of $\overline{\Delta T}$. However, for the uniform wall temperature solutions, and for the highest temperature differences used, the uniform viscosity solutions, with viscosity taken at the reference temperature, are quite accurate. For both free and forced convection, the largest error in the uniform viscosity solutions is only about 3%, overestimating in the former case and underestimating in the latter. The variable viscosity

Table 1. Effect of variable condensate viscosity on mean heat flux for condensation of ethylene glycol on a horizontal tube ($d = 12.5$ mm, $A = 0.6$)

Free convection $T_v = 425$ K				Forced convection ($U_\infty = 100$ m s^{-1}) $T_v = 380$ K				
$\overline{\Delta T}$ (K)	†	$\bar{q}$ (kW m^{-2}) ‡	§	$\overline{\Delta T}$ (K)	†	$\bar{q}$ (kW m^{-2}) ‡	§	A
40	122.0	120.6	120.5	20	289.6	290.9	291.0	0.0
			123.3				301.1	0.2
			126.0				314.9	0.4
			128.7				332.9	0.6
70	171.7	167.7	167.8	40	500.5	506.3	505.5	0.0
			175.8				539.3	0.2
			183.4				580.7	0.4
			190.9				631.2	0.6
100	204.1	197.5	197.6	60	636.3	653.2	651.8	0.0
			212.9				719.6	0.2
			227.8				801.3	0.4
			242.4				897.3	0.6

† Uniform viscosity, uniform wall temperature, viscosity at T^*.
‡ Variable viscosity, uniform wall temperature.
§ Variable viscosity, variable wall temperature.

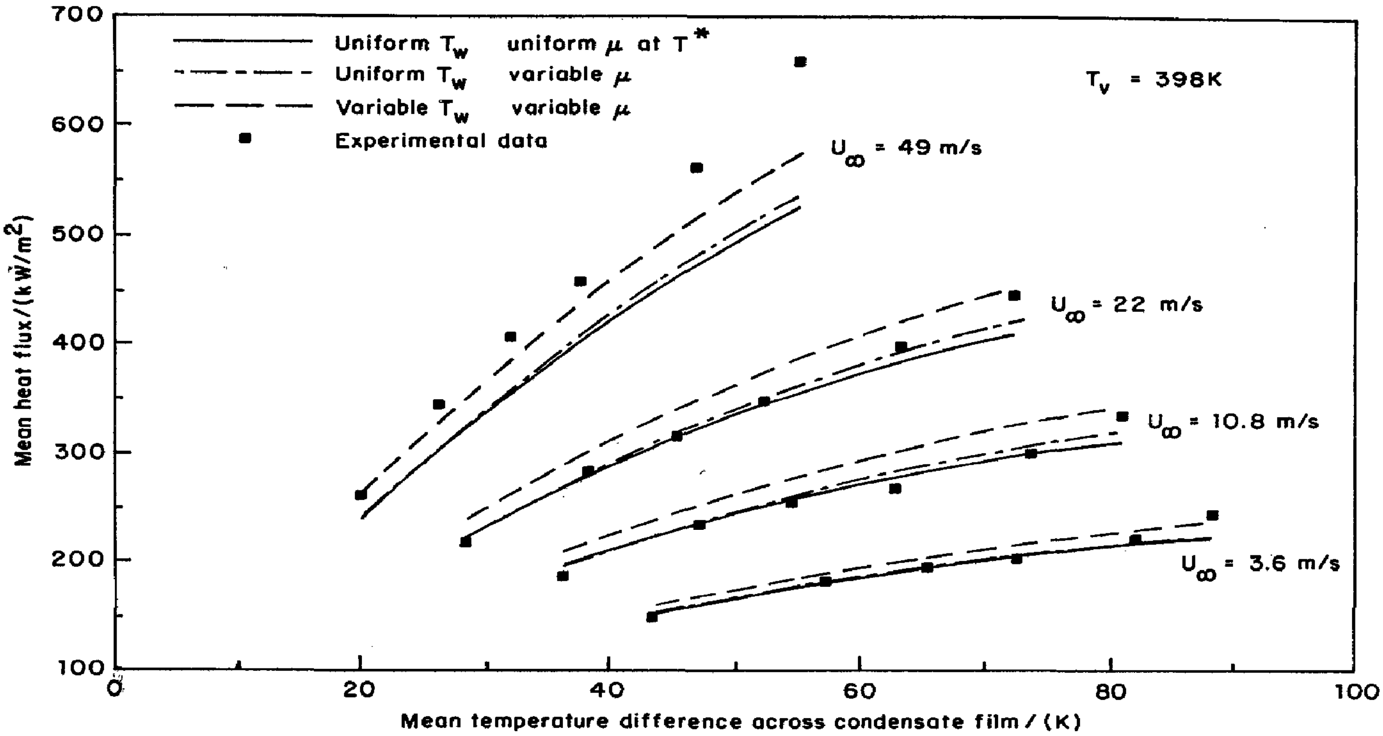

FIG. 3. Comparison of variable wall temperature and variable viscosity theory with experimental data.

results are seen to be markedly affected by the value of A, the mean heat flux increasing with increase in wall temperature variation. At the highest values of $\overline{\Delta T}$ and A ($=0.6$), the error in the uniform wall temperature, uniform viscosity results is 16% (at $\overline{\Delta T} = 100$ K) for free convection and 29% (at $\overline{\Delta T} = 60$ K) for forced convection.

Figure 3 shows comparisons with experimental data [15, 16]. The measurements were made using a horizontal copper condenser tube of 12.5 mm dia. with vertically downward flowing ethylene glycol vapor. The vapor velocity, temperature and pressure, the mean heat flux and the tube wall temperature at six angular locations, were accurately measured. Values of A were found by fitting the measured circumferential temperature distributions to equation (3). For the data shown in Fig. 3, A varied from 0.05 at the lowest to 0.24 at the highest vapor velocity.

As noted earlier, Fig. 3 shows the relatively small effect of taking account of radial viscosity variation only, i.e. using a variable viscosity solution with uniform wall temperature. When circumferential temperature variation is included, significantly higher heat fluxes are obtained.

The fact that variable wall temperature solutions overestimate the measured heat flux at lower vapor velocities is due to the shear stress approximation used in the analysis, and in particular the fact that this takes no account of vapor boundary layer separation. A more accurate representation of the surface shear stress, such as that given by the method of Truckenbrodt [17] and used by Honda and Fujii [7], would give somewhat lower mean heat fluxes [11], but should not invalidate the conclusions regarding the effects of circumferential and radial variation of viscosity.

CONCLUSIONS

1. Circumferential viscosity variation has a stronger effect on heat transfer in condensation on a horizontal tube than does previously considered radial variation.
2. When account is taken of circumferential viscosity variation, higher heat fluxes (and hence higher mean heat-transfer coefficients) than those found when using uniform viscosity (evaluated at a reference temperature determined on the basis of radial property variation only) are found. The uniform viscosity reference temperature calculation is therefore conservative.
3. The effect of circumferential viscosity variation becomes more important as the surface temperature becomes more non-uniform. This, in turn, occurs when the vapor-side resistance is a smaller proportion of the overall resistance. Thus, when the effect of variable viscosity is strongest, its effect on the overall heat-transfer coefficient is weakest. For the purposes of calculating overall coefficients, the reference temperature method should generally therefore not be greatly in error, as well as being conservative.

REFERENCES

1. D. A. Labuntsov, Effect of temperature dependence of physical parameters of condensate heat transfer in film condensation of steam, *Teploenergetika* 4, 49–51 (1957).

2. W. J. Minkowycz and E. M. Sparrow, Condensation heat transfer in the presence of noncondensables, interfacial resistance, superheating, variable properties and diffusion, *Int. J. Heat Mass Transfer*, **9**, 1125–1144 (1966).
3. E. M. Sparrow, W. J. Minkowycz and M. Saddy, Forced convection condensation in the presence of non-condensables and interfacial resistance, *Int. J. Heat Mass Transfer* **10**, 1829–1845 (1967).
4. G. Poots and R. G. Miles, Effects of variable physical properties on laminar film condensation of saturated steam on a vertical flat place, *Int. J. Heat Mass Transfer* **10**, 1677–1692 (1967).
5. V. E. Denny and A. F. Mills, Nonsimilar solutions for laminar film condensation on a vertical surface, *Int. J. Heat Mass Transfer* **12**, 965–979 (1969).
6. T. Fujii, H. Honda and K. Oda, Condensation of steam on a horizontal tube—the influence of oncoming velocity and thermal condition at the tube wall. In *Condensation Heat Transfer*, Proceedings of 18th National Heat Transfer Conference, San Diego, pp. 35–43 (1979).
7. H. Honda and T. Fujii, Condensation of flowing vapour on a horizontal tube—numerical analysis on a conjugate heat transfer problem, *Trans. ASME, J. Heat Transfer* **106**, 841–848 (1984).
8. S. Rahbar and J. W. Rose, New measurements for forced-convection film condensation, *Proceedings of* 1*st National Heat Transfer Conference*, Institute Chemical Engineers Symposium **86**, Vol. 1, pp. 619–632 (1984).
9. W. C. Lee, S. Rahbar and J. W. Rose, Film condensation of refrigerant 113 and ethylene glycol on a horizontal tube—effect of vapour velocity, *Trans. ASME, J. Heat Transfer* **106**, 524–530 (1984).
10. S. B. Memory and J. W. Rose, Free convection laminar film condensation on a horizontal tube with variable wall temperature, *Int. J. Heat Mass Transfer* **34**, 2775–2778 (1991).
11. S. B. Memory, W. C. Lee and J. W. Rose, Forced convection film condensation on a horizontal tube—effect of surface temperature variation, *Int. J. Heat Mass Transfer* **36**, 1671–1676 (1993).
12. I. G. Shekriladze and V. I. Gomelauri, Theoretical study of laminar film condensation of a flowing vapour, *Int. J. Heat Mass Transfer* **9**, 581–591 (1966).
13. W. Nusselt, Die Oberflachenkondensation des Wasserdampfes, *ZVDI* **60**, 541–546, 569–575 (1916).
14. R. W. Gallant, *Physical Properties of Hydrocarbons*, Vol. 1, pp. 109–123. Gulf, Houston, TX (1968).
15. S. B. Memory and J. W. Rose, Film condensation of ethylene glycol on a horizontal tube at high vapor velocity, *Proceedings of* 8*th International Heat Transfer Conference*, San Francisco, Vol. 4, pp. 1607–1612 (1986).
16. S. B. Memory, Forced convection film condensation on a horizontal tube at high vapour velocity, Ph.D. Thesis, University of London (1989).
17. E. Truckenbrodt, Ein einfaches Naherungsverfahren zum Berechnen der laminaren Riebungsicht mit Absangung, *Forschung. Ing. Wes.* **22**, 147–157 (1956).